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# Excitation of $\text{Ho}^{3+}$ ions via energy transfers from $\text{Cr}^{3+}$ and $\text{Tm}^{3+}$ ions in (Ca, Zr)-substituted $\text{Gd}_3\text{Ga}_5\text{O}_{18}$ single crystals

A Brenier, C Madej, C Pedrini and G Boulon

Laboratoire de Physico-Chimie des Matériaux Luminescents (Unité de Recherche associée au CNRS 442), Université Lyon I, 43 boulevard du 11 Novembre 1918, 69622 Villeurbanne, France

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**Abstract.** (Ca, Zr)-substituted  $\text{Gd}_3\text{Ga}_5\text{O}_{12}$  single crystals containing various amounts of  $\text{Cr}^{3+}$ ,  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions were grown using the Czochralski method. The energy transfers from  $\text{Cr}^{3+}$  and  $\text{Tm}^{3+}$  ions were analysed and their quantum efficiencies calculated. The stimulated emission cross section of the  $^5\text{I}_7$  level of  $\text{Ho}^{3+}$  in the 1800–2200 nm region was determined using McCumber's theory.

## 1. Introduction

Eye-safe 2  $\mu\text{m}$  infrared solid state lasers are usually obtained from the  $^5\text{I}_7 \rightarrow ^5\text{I}_8$  transition of  $\text{Ho}^{3+}$  impurity in different fluorides, garnets and other oxides. The fluorescence of the  $\text{Ho}^{3+}$  ions can be sensitized by erbium and thulium impurities in fluorides and by chromium and thulium ions in garnets. In previous papers [1, 2],  $\text{Cr}^{3+} \rightarrow \text{Tm}^{3+}$  and  $\text{Tm}^{3+} \rightarrow \text{Tm}^{3+}$  energy transfers were investigated in detail in (Ca, Zr)-substituted  $\text{Gd}_3\text{Ga}_5\text{O}_{12}$  crystals. The results of this analysis are used in the present study of the direct  $\text{Tm}^{3+} \rightarrow \text{Ho}^{3+}$  energy transfer and of the  $\text{Cr}^{3+} \rightarrow \text{Ho}^{3+}$  transfer process via  $\text{Tm}^{3+}$  ions. The quantum yield of the  $^4\text{T}_2(\text{Cr}^{3+}) \rightarrow ^5\text{I}_7(\text{Ho}^{3+})$  transfer and the stimulated emission cross section of the  $^5\text{I}_7(\text{Ho}^{3+})$  level at around 2  $\mu\text{m}$  were deduced. Such information is of particular interest for laser applications.

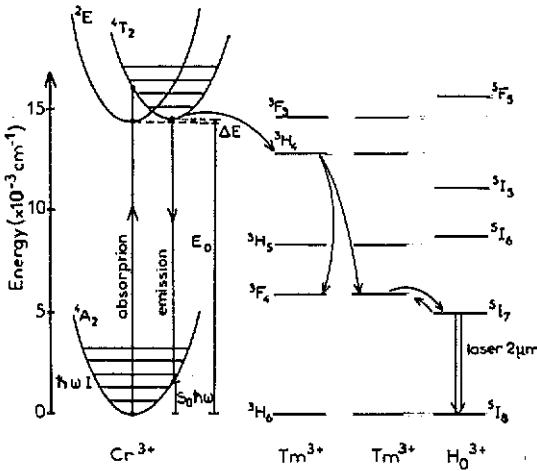
The experimental techniques used for optical studies and for crystal growth are not described here since they are the same as reported in our previous work [1]. We have grown three crystals by the Czochralski method, containing a nearly constant amount of  $\text{Cr}^{3+}$  ions and various contents of  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  ions (table 1). Chromium ions occupy  $\text{Ga}^{3+}$  octahedral sites and rare-earth impurities replace  $\text{Gd}^{3+}$  ions. The segregation coefficient is 2.8 for  $\text{Cr}^{3+}$  and 1.2 for  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$ .

## 2. Excited-state dynamics and energy transfers

The general excited-state dynamics of the triple-doped system is sketched in figure 1, where all the energy levels and the processes considered in the present study are indicated.

**Table 1.** Rare-earth ion concentrations of the samples used for the optical studies, wherein Gd is replaced by  $\text{Gd}_{1-x-y}\text{Tm}_x\text{Ho}_y$ .

$x$ (at. %)	$y$ (at. %)	$\text{Cr}^{3+}$ ( $10^{20}$ ions $\text{cm}^{-3}$ )	$\text{Tm}^{3+}$ ( $10^{20}$ ions $\text{cm}^{-3}$ )	$\text{Ho}^{3+}$ ( $10^{20}$ ions $\text{cm}^{-3}$ )
1	0.5	0.64	1.28	0.60
7	0.5	0.78	9.31	0.53
1	2	0.76	1.22	2.21



**Figure 1.** Energy level scheme for the impurity ions and main energy transfer processes.

The addition of 0.5 at. % Ho to samples doped with 1 or 7 at. % Tm does not change the decay of the  ${}^3\text{H}_4$  emission significantly. When we add 2 at. % Ho, a weak decrease in the decay time of the  ${}^3\text{H}_4 \rightarrow {}^3\text{F}_4$  transition is observed, indicating the existence of a weak  ${}^3\text{H}_4(\text{Tm}^{3+}) \rightarrow \text{Ho}^{3+}$  energy transfer. Knowing the microscopic constant  $P$  of the  ${}^3\text{H}_4(\text{Tm}^{3+}), {}^3\text{H}_6(\text{Tm}^{3+}) \rightarrow {}^3\text{F}_4(\text{Tm}^{3+}), {}^3\text{F}_4(\text{Tm}^{3+})$  cross relaxation process, deduced in [1] from the Yokota–Tanimoto [3] equation when the  $\text{Ho}^{3+}$  ions are absent ( $C_{\text{DA}} \equiv P = 1.3 \times 10^{-39} \text{ cm}^6 \text{ s}^{-1}$ ), we have calculated the microscopic constant  $P'$  of the  ${}^3\text{H}_4(\text{Tm}^{3+}) \rightarrow \text{Ho}^{3+}$  transfer under  ${}^3\text{H}_4$  excitation using the same formula:

$$N(t) = N_0 \exp\{-t/\tau - bt^{1/2}[(1 + 10.87x + 15.50x^2)/(1 + 8.743x)]^{3/4}\} \quad (1)$$

in which the contribution of the  $\text{Tm}^{3+} \rightarrow \text{Ho}^{3+}$  transfer was added:

$$b = \frac{4}{3}\pi^{3/2}(C_{\text{Tm}}\sqrt{P} + C_{\text{Ho}}\sqrt{P'}). \quad (2)$$

$P'$  was found to be 70 times weaker than  $P$ .

Under  $\text{Cr}^{3+}$  excitation, a weak  ${}^4\text{T}_2(\text{Cr}^{3+}) \rightarrow \text{Ho}^{3+}$  energy transfer was also observed and, using the previous procedure, its microscopic constant was calculated to be 14 times less than that ( $7.7 \times 10^{-39} \text{ cm}^6 \text{ s}^{-1}$ ) characterizing the  ${}^4\text{T}_2(\text{Cr}^{3+}) \rightarrow \text{Tm}^{3+}$  transfer [1].

Let us now consider the energy transfer occurring from the  ${}^3\text{F}_4$  lowest excited state of  $\text{Tm}^{3+}$  to the  ${}^5\text{I}_7$  lowest excited state of  $\text{Ho}^{3+}$ . This transfer is very efficient and can be studied after either  ${}^3\text{H}_5$  or  ${}^3\text{F}_4$  excitation of  $\text{Tm}^{3+}$  ions, since the first excitation is followed

by a very fast (less than 1  $\mu s$ )  ${}^3H_5 \rightarrow {}^3F_4$  relaxation and leads to an exponential  ${}^3F_4 \rightarrow {}^3H_6$  emission (about 1.8  $\mu m$ ) with no rise time in the (Cr, Tm) co-doped crystals [1]. The introduction of  $Ho^{3+}$  ions is responsible for the non-exponential faster decay of this fluorescence and simultaneously gives rise to a strong 2  $\mu m$   ${}^5I_7 \rightarrow {}^5I_8$  emission transition of the  $Ho^{3+}$  ions. In order to describe the  ${}^3F_4(Tm^{3+}) \rightarrow {}^5I_7(Ho^{3+})$  transfer, it is very convenient to follow Chandrasekhar's [4] procedure described in the appendix of [1] when applied to the case of energy transfers.

The application of Markov's method to this problem leads, in the case of dipole-dipole interaction, to a simple expression for the stationary distribution  $\omega(\varphi)$  of the transfer rate of a given donor ion D to have a prescribed value  $\varphi$ :

$$\omega(\varphi) = (b/2\pi^{1/2})\varphi^{-3/2} \exp(-b^2/4\varphi) \tag{3}$$

with

$$b = \frac{4}{3}\pi^{3/2} C_A \sqrt{P} \tag{4}$$

and

$$\varphi = \sum_{N_A} T_{DA_i}(r_i). \tag{5}$$

$C_A$  is the concentration of acceptor ions  $A_i$ ,  $P \equiv C_{DA}$  is the microscopic constant of the  $D \rightarrow A$  transfer,  $N_A$  is the total number of acceptor ions,  $r_i$  is the distance between D and  $A_i$  ions and  $T_{DA_i}$  represents the transfer rate on each acceptor  $A_i$ .

To describe the transfers, it is therefore correct to use the rate equations for a class  $\varphi$  of ions and their complete resolution is possible, knowing  $\omega(\varphi)$ . In simple cases, the resolution leads to the well known Inokuti-Hirayama [5] equation. However, the method is of most interest in more complicated cases where the Inokuti-Hirayama formula is no longer valuable.

Returning to the particular case of the  ${}^3F_4(Tm^{3+}) \rightarrow {}^5I_7(Ho^{3+})$  energy transfer, the population  $N_\varphi(t)$  of  $Tm^{3+}$  ions belonging to the class  $\varphi$  and in the  ${}^3F_4$  excited state decays according to the rate equation

$$\dot{N}_\varphi(t) = -N_\varphi(t)(1/\tau + \varphi) \tag{6}$$

where  $\tau$  represents the lifetime of  ${}^3F_4$  without any  $Ho^{3+}$  ions. The solution of (6) is obvious:

$$N_\varphi(t) = N_\varphi^0 \exp[-t(1/\tau + \varphi)] \tag{7}$$

with

$$N_\varphi^0 = N^0 \omega(\varphi) \quad \text{at } t = 0. \tag{8}$$

$N^0$  is the overall initial population of the  ${}^3F_4$  level. The total population  $N(t)$  of the  ${}^3F_4$  level is then

$$N(t) = \int_0^\infty N_\varphi(t) d\varphi \tag{9}$$

and, taking into account (3), we find that

$$N(t) = N^0 \exp(-t/\tau - b\sqrt{t}) \tag{10}$$

with

$$b = \frac{4}{3}\pi^{3/2} C_{Ho} \sqrt{P''}. \tag{11}$$

$C_{Ho}$  is the concentration of  $Ho^{3+}$  in the ground state and  $P''$  is the microscopic constant of the  ${}^3F_4 \rightarrow {}^5I_7$  transfer.

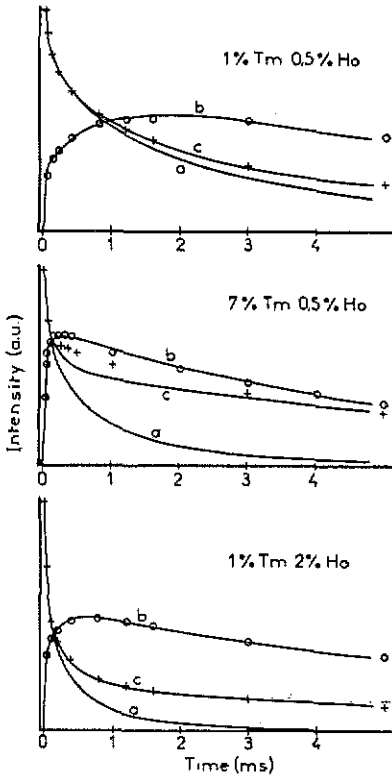


Figure 2. Experimental decays: +,  ${}^3F_4 \rightarrow {}^3H_6$  fluorescence of  $Tm^{3+}$ ; O,  ${}^5I_7 \rightarrow {}^5I_8$  fluorescence of  $Ho^{3+}$ ; —, best fits to experimental data obtained with the models described in the text.

As previously predicted, in this simple case, equation (10) is simply the Hinokuti-Hirayama formula. It fits the beginning of the experimental  ${}^3F_4 \rightarrow {}^3H_6$  fluorescence (first 100  $\mu s$  in figure 2 curves a) quite well. However, the rest of the decay is not correctly described. In fact, in  $YLiF_4$  and in indium-based fluoride glasses, we had clearly identified an efficient  ${}^5I_7(Ho^{3+}) \rightarrow {}^3F_4(Tm^{3+})$  back transfer [6, 7]. The final parts of the decay of both the  ${}^3F_4$  and the  ${}^5I_7$  levels are similar (figure 2), indicating that such a mechanism actually occurs in the samples at present studied and it has to be included in equation (10). The back transfer is proportional to the number  $\omega(\varphi)$  of  $Tm^{3+}$  ions of the class  $\varphi$  and to the total number  $N'(t)$  of  $Ho^{3+}$  ions in the  ${}^5I_7$  excited state, so that equation (6) is replaced by

$$\dot{N}_\varphi(t) = -N_\varphi(t)(1/\tau + \varphi) + k\omega(\varphi)N'(t) \quad (12)$$

where  $k$  is the  $\varphi$ -independent back-transfer constant.  $N'(t)$  is given by a rate equation analogous to (12) and it turns out that two coupled equations have to be solved. In order to avoid the difficult resolution of such a system, it is better to proceed in the following way. A preliminary fit of  $N'(t)$  to the experimental data is first obtained with the expression

$$N'(t) \approx A_1[\exp(-t/\tau') - \exp(-t/\tau_1)] + A_2[\exp(-t/\tau') - \exp(-t/\tau_2)]. \quad (13)$$

$\tau'$  ( $=9.4$  ms) is the lifetime of  ${}^5I_7$  measured after direct excitation of  $Ho^{3+}$ ;  $A_1$  and  $\tau_1$  are

**Table 2.**  ${}^3\text{F}_4(\text{Tm}^{3+}) \rightarrow {}^5\text{I}_7(\text{Ho}^{3+})$  transfer parameters.

Crystal	$P''$ ( $10^{-39} \text{ cm}^6 \text{ s}^{-1}$ )	$\tau$ ( $10^{-3} \text{ s}$ )	$\eta_T$ (%)
1 at. % Tm; 0.5 at. % Ho	3.0	11.25	52
7 at. % Tm; 0.5 at. % Ho	14.3	6.0	35.5
1 at. % Tm; 2 at. % Ho	2.1	11.25	—

adjustable parameters without real physical meaning. The best fits lead to curves b in figure 2. The solution of equation (12) is then

$$\begin{aligned}
 N_{\varphi}(t) = N^0 \omega(\varphi) \exp\left(-\frac{t}{\tau} - t\varphi\right) + k\omega(\varphi) \left\{ \frac{A_1 + A_2}{1/\tau + \varphi - 1/\tau'} \right. \\
 \times \left[ \exp\left(-\frac{t}{\tau'}\right) - \exp\left(-\frac{t}{\tau - t\varphi}\right) \right] \\
 \left. - \sum_{i=1}^2 \frac{A_i}{1/\tau + \varphi - 1/\tau_i} \left[ \exp\left(-\frac{t}{\tau'}\right) - \exp\left(-\frac{t}{\tau_i} - t\varphi\right) \right] \right\}. \quad (14)
 \end{aligned}$$

The total population  $N(t)$  of the  ${}^3\text{F}_4$  level is given after integration over  $\varphi$ :

$$N(t) = N^0 \exp(-t/\tau - b\sqrt{t} + I) \quad (15)$$

where the first term is the Hinokuti–Hirayama expression and  $I$  is the result of a numerical integration of the second term of (14) over  $\varphi$  for each value of  $t$ .

Equation (15) leads to curves c in figure 2. The best fits were obtained for values of  $P''$  indicated in table 2 ( $P''$  is the microscopic constant of the  ${}^3\text{F}_4(\text{Tm}) \rightarrow {}^5\text{I}_7(\text{Ho})$  transfer). Note that  $P''$  is the only free physical parameter of the model because the intrinsic lifetime  $\tau$  of  ${}^3\text{F}_4$  was separately measured in samples free of  $\text{Ho}^{3+}$  ions [1]. It should be remarked that, for samples containing 1 at. % Tm, the best fits were obtained with comparable values of  $P''$  and describe the experimental results well. On the other hand, in the case of the heavily Tm-doped sample (7 at. %), the best fit is less satisfactory and was obtained with a much higher value of  $P''$ . This is probably because our model does not take into account the energy diffusion between the  $\text{Tm}^{3+}$  ions which is certainly effective in the case of high concentrated compounds.

Table 2 also gives the quantum yield  $\eta_T$  of the  $\text{F}_4 \rightleftharpoons {}^5\text{I}_7$  transfer when the  ${}^5\text{I}_7$  population is a maximum. It was deduced experimentally (and not from the theoretical model) for the two samples containing the same amount of Ho (0.5 at. %) and different Tm concentrations (1 and 7 at. %). It is clear that the back transfer induces a strong decrease (about 30%) in the quantum yield in going from 1 to 7 at. % Tm.

### 3. Quantum yield of the ${}^4\text{T}_2(\text{Cr}^{3+}) \rightarrow {}^5\text{I}_7(\text{Ho}^{3+})$ transfer

The schematic representation of the absorbed energy conversion in (Ca, Zr)-substituted  $\text{Gd}_3\text{Ga}_5\text{O}_{12}$  crystals (figure 1) shows that the  ${}^4\text{T}_2(\text{Cr}^{3+}) \rightarrow {}^5\text{I}_7(\text{Ho}^{3+})$  transfer takes place

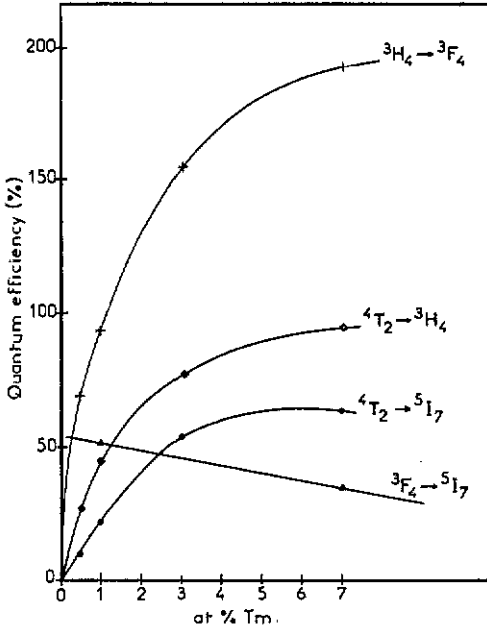


Figure 3. Quantum yields of various transfers in samples containing 0.5 at. % Cr<sup>3+</sup> and Ho<sup>3+</sup> and different amounts of Tm<sup>3+</sup>. The concentrations are nominal.

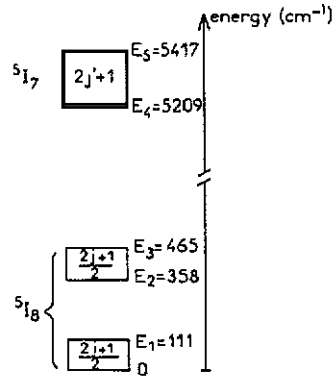


Figure 4. Experimental Stark energy levels of the ⁵I<sub>8</sub> ground state and the ⁵I<sub>7</sub> lowest excited state of Ho<sup>3+</sup>.

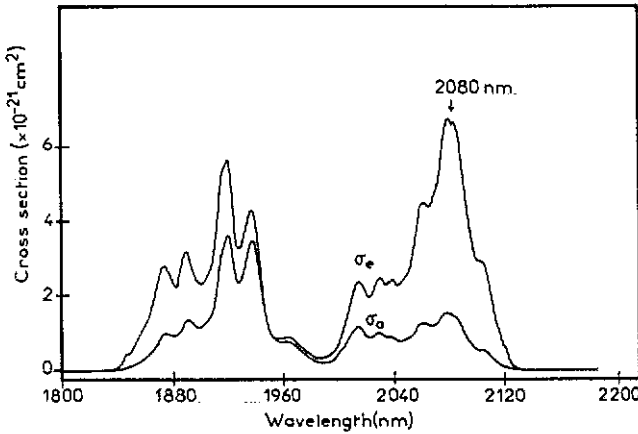


Figure 5. Absorption and stimulated emission cross sections of the ⁵I<sub>7</sub>(Ho<sup>3+</sup>) excited state at room temperature.

in three stages: ⁴T<sub>2</sub>(Cr<sup>3+</sup>) → ³H<sub>4</sub>(Tm<sup>3+</sup>) energy transfer, ³H<sub>4</sub>(Tm<sup>3+</sup>) → ³F<sub>4</sub>(Tm<sup>3+</sup>) cross-relaxation exchange of excitation between Tm<sup>3+</sup> ions and ³F<sub>4</sub>(Tm<sup>3+</sup>) → ⁵I<sub>7</sub>(Ho<sup>3+</sup>) transfer which can be preceded by energy migration between Tm<sup>3+</sup> ions. The quantum yields of the steps were calculated for samples of constant Cr<sup>3+</sup> and Ho<sup>3+</sup> concentrations

(0.5 at.%) and containing different amounts of  $Tm^{3+}$  ions. The related curves are displayed in figure 3. The efficiency of the first process  ${}^4T_2 \rightarrow {}^3H_4$  was reported in a previous paper [1]. A good description of the  ${}^3H_4 \rightarrow {}^3F_4$  de-excitation of the  $Tm^{3+}$  ions must include the cross relaxation type of energy transfer between the  $Tm^{3+}$  ions, the  $Tm$  intra-centre de-excitation and the  ${}^3H_4(Tm^{3+}) \rightarrow Ho^{3+}$  direct energy transfer [6]. In order to take into account all these phenomena, the quantum yield of the second  ${}^3H_4 \rightarrow {}^3F_4$  mechanism was evaluated by measuring the relative  ${}^3F_4$  emission intensities obtained after excitations of  ${}^3H_4$  and  ${}^3F_4$ , for samples placed inside an integrating sphere. Note that the value of the quantum rate at high  $Tm$  concentration is nearly 2 since one  $Tm^{3+}$  ion in its  ${}^3H_4$  excited state may lead to two  $Tm^{3+}$  ions in the  ${}^3F_4$  state. The efficiency of the third  ${}^3F_4 \rightarrow {}^5I_7$  step was determined in the previous section. Since we have only two experimental points, we have assumed a linear dependence of the quantum yield with  $Tm$  concentration. The product of the efficiencies of the three stages finally gives the quantum yield of the  ${}^4T_2(Cr^{3+}) \rightarrow {}^5I_7(Ho^{3+})$  energy transfer. The related curve (figure 3) exhibits a maximum for 5 at.%  $Tm$  concentration. This result is important since it shows that, because of the  ${}^5I_7(Ho^{3+}) \rightarrow {}^3F_4(Tm^{3+})$  back transfer, a further increase in  $Tm^{3+}$  concentration is not efficient.

#### 4. Stimulated emission cross section of the ${}^5I_7(Ho^{3+})$ level around 2 $\mu m$

The calculation of the gain per unit length of a pumped laser material requires knowledge of the absorption cross section  $\sigma_a(\lambda)$  and of the stimulated emission cross-section  $\sigma_e(\lambda)$  at the wavelength  $\lambda$ . The latter is given by McCumber's [8] relation

$$\sigma_e(\lambda) = [N^0({}^5I_8)/N^0({}^5I_7)]\sigma_a(\lambda) \exp[-E(\lambda)/kT]. \tag{16}$$

$E(\lambda)$  is the energy of one photon at the wavelength  $\lambda$ , and  $N^0({}^5I_8)$  and  $N^0({}^5I_7)$  are the populations of the  ${}^5I_8$  and  ${}^5I_7$  levels at thermal equilibrium.

The absorption and emission spectra recorded at liquid-helium temperature lead to the energy level scheme of  ${}^5I_8$  and  ${}^5I_7$  Stark levels displayed in figure 4. The ground state can be represented by two thick levels and the excited state by one thick level. The limiting energies  $E_i$  of each thick level  $i$  are also indicated in the figure. As previously shown [7], the density of the statistical weight  $g_i$  of a level  $i$  can be evaluated as

$$g_i = d_i/(E_{ih} - E_{il}) \tag{17}$$

where  $d_i$  stands for the  $J$  degeneracy, and  $E_{ih}$  and  $E_{il}$  are the highest and lowest energies of the thick level  $i$ , respectively:  $g = (2J' + 1)/(E_5 - E_4)$  for the  ${}^5I_7$  level;  $g = (2J + 1)/2E_1$  for the lowest thick sublevel of  ${}^5I_8$ ;  $g = (2J + 1)/2(E_3 - E_2)$  for the highest thick sublevel of  ${}^5I_8$ .

Finally, the Boltzmann law leads to

$$N^0({}^5I_8) \propto kT\{[(2J + 1)/2E_1][1 - \exp(-E_1/kT)] + [2J + 1/2(E_3 - E_2)][\exp(-E_2/kT) - \exp(-E_3/kT)]\} \tag{18}$$

$$N^0({}^5I_7) \propto kT\{(2J' + 1)/(E_5 - E_4)[\exp(-E_4/kT) - \exp(-E_5/kT)]\}$$

with  $J = 8$  and  $J' = 7$ .

Figure 5 shows the curves of  $\sigma_a(\lambda)$  and  $\sigma_e(\lambda)$  at room temperature. The stimulated emission cross section exhibits a maximum at 2080 nm. Therefore a laser effect from



Ho<sup>3+</sup> ions in (Ca, Zr)-substituted Gd<sub>3</sub>Ga<sub>5</sub>O<sub>12</sub> is expected at around this wavelength. Our simple model of Stark structure of the <sup>5</sup>I<sub>7</sub> and <sup>5</sup>I<sub>8</sub> levels predicts a radiative lifetime of <sup>5</sup>I<sub>7</sub> at around 10.1 ms. This value is close to the experimental decay time of 9.4 ms for the <sup>5</sup>I<sub>7</sub> level, meaning that non-radiative losses in the <sup>5</sup>I<sub>7</sub> → <sup>5</sup>I<sub>8</sub> transition are rather low.

## 5. Concluding remarks

This work shows that efficient energy transfers occur in (Ca, Zr)-substituted Gd<sub>3</sub>Ga<sub>5</sub>O<sub>12</sub> garnet from Cr<sup>3+</sup> to Ho<sup>3+</sup> impurity ions via Tm<sup>3+</sup> ions. Optimized systems are obtained with impurity ion concentrations of 0.5 at. % for Cr<sup>3+</sup> and Ho<sup>3+</sup> and of 5 at. % for Tm<sup>3+</sup> owing to the occurrence of Ho<sup>3+</sup> → Tm<sup>3+</sup> back transfer. Like Gd<sub>3</sub>Sc<sub>2</sub>Ga<sub>3</sub>O<sub>12</sub> laser material, this garnet could be used to obtain a low threshold 2 μm laser, but at a much lower cost.

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